Zoom Class 7/5/2020

Properties & Substitution Problems:

Ques Evaluate $\int_2^3 \frac{1}{x \log x} dx$

Soln $I = \int_2^3 \frac{1}{x \log x} dx$.

Put $\log x = y \Rightarrow \frac{1}{x} dx = dy$

Also when $x = 2$, $y = \log 2$
when $x = 3$, $y = \log 3$



$\therefore I = \int_{\log 2}^{\log 3} \frac{1}{y} dy$

$= \left[\log y\right]_{\log 2}^{\log 3}$

$= \log(\log 3) - \log(\log 2)$

$= \log\left[\frac{\log 3}{\log 2}\right]$

Ans

Ques Evaluate $\int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx$

Soln $I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx$ —— (1)

$\Rightarrow I = \int_0^{\pi/2} \frac{\sqrt{\cos(\pi/2 - x)}}{\sqrt{\cos(\pi/2-x)}+\sqrt{\sin(\pi/2-x)}}\,dx$ —— using property (4)

$\Rightarrow I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$ —— (2)

Adding (1) & (2) we get



$2I = \int_0^{\pi/2} \left\{ \frac{\sqrt{\cos x}+\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}} \right\} dx$

$\Rightarrow 2I = \int_0^{\pi/2} dx \Rightarrow 2I = \left[ x \right]_0^{\pi/2}$

$\Rightarrow I = \frac{1}{2}\left[\frac{\pi}{2} - 0\right] = \frac{\pi}{4}$ Ans

Ques Evaluate $\int_{-1}^{1} |x|\,dx$



Soln $\because |x| = |-x|$ it is an even function

$\therefore I = \int_{-1}^{1} |x|\,dx = 2\int_{0}^{1} |x|\,dx = 2\left[\frac{x^2}{2}\right]_0^1$

$= 2\left[\frac{1}{2} - 0\right] = 2 \times \frac{1}{2} = 1$ Ans

Ques Evaluate $\int_1^{\sqrt{e}} x \log x \, dx$.

Soln $I = \int_1^{\sqrt{e}} x \log x \, dx$

Integrating by parts, we have

$$I = \left[\frac{1}{2}x^2 \log x - \frac{1}{2}\int x\,dx\right]_1^{\sqrt{e}}$$

$$= \frac{1}{2}\left[x^2 \log x - \frac{x^2}{2}\right]_1^{\sqrt{e}}$$

$$= \frac{1}{2}\left[e \log\sqrt{e} - \frac{e}{2} - \log 1 + \frac{1}{2}\right]$$

$$= \frac{1}{2}\left[\frac{e}{2}\log e - \frac{e}{2} - \log 1 + \frac{1}{2}\right]$$

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$$= \frac{1}{2}\left[\frac{e}{2}\log e - \frac{e}{2} + \frac{1}{2}\right]$$

$$= \frac{e}{4}\log e - \frac{e}{4} + \frac{1}{4}$$

$$= \cancel{\frac{e}{4}} - \cancel{\frac{e}{4}} + \frac{1}{4}$$

$$= \frac{1}{4}$$ Ans

**Ques** Evaluate $\displaystyle\int_0^{\pi/2} \frac{dx}{a^2\cos^2 x + b^2\sin^2 x}$

**Soln** $\displaystyle I = \int_0^{\pi/2} \frac{dx}{a^2\cos^2 x + b^2\sin^2 x} = \int_0^{\pi/2} \frac{\sec^2 x\, dx}{a^2 + b^2\tan^2 x}$

$$= \frac{1}{b^2}\int_0^{\pi/2} \frac{\sec^2 x\, dx}{\left(\frac{a}{b}\right)^2 + (\tan x)^2}$$

Put $\tan x = y \Rightarrow \sec^2 x\, dx = dy$

Keeping orignal limits we get

$$I = \frac{1}{b^2}\int_0^{\pi/2} \frac{dy}{(a/b)^2 + (y)^2}$$



$$\cancel{= \frac{1}{b^2}\left[\tan^{-1}\left\{\frac{b}{a}\tan x\right\}\right]_0^{\pi/2} dx}$$

$$= \frac{1}{b^2}\left[\frac{b}{a}\tan^{-1}\left\{\frac{y}{a/b}\right\}\right]_0^{\pi/2} = \frac{1}{b^2}\times\frac{b}{a}\left[\tan^{-1}\left(\frac{b}{a}\tan x\right)\right]_0^{\pi/2}$$

$$= \frac{1}{ab}\left[\tan^{-1}\left(\frac{b}{a}\tan\frac{\pi}{2}\right) - \tan^{-1}\left(\frac{b}{a}\tan 0\right)\right]$$

$$= \frac{1}{ab}\left[\tan^{-1}(\infty) - \tan^{-1}(0)\right]$$

$$= \frac{1}{ab}\left[\frac{\pi}{2} - 0\right] = \frac{\pi}{2ab}$$

**Ans**